Det Kgl. Danske Videnskabernes Selskab.
Mathematisk-fysiske Meddelelser. **XI**, 8.

TABLE GIVING $\tan\frac{v}{2}$ IN PARABOLIC MOTION WITH ARGUMENT $M = (t - T)q^{-3/2}$ FROM $M = 275$ TO $M = 4515$.

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BIANCO LUNOS BOGTRYKKERI A/S
1932

The substitution of logarithms by calculating machines has been considerably hampered through the lack of tables adapted to the new methods of computation, the absence of even ordinary (non logarithmic) trigonometrical tables has been and is still partly felt. But even more the absence of tables has been felt at those domains, where the calculating machine has demanded new methods of computation. We have a typical example in the computation of an ephemeris for a parabolic orbit. Using logarithms the true anomaly v is found by the aid of Barker's table, and then the coordinates in the plane of the orbit $r \cos v$ and $r \sin v$ are computed; but with the calculating machine the determining of v is completely avoided, and a simplification of the computation is obtained through the direct determination of $\tan\frac{v}{2}$ with $M = \frac{t-T}{q^{3/2}}$ as an argument, and the subsequent computation of the coordinates in the plane of the orbit written as $q\left(1-\tan^2\frac{v}{2}\right)$ and $2\,q\tan\frac{v}{2}$.

The first table of this kind was computed already in 1927 by B. Strömgren (Memoirs of the British Astronomical Association vol. XXVII part 2, and Publ. of the Copenhagen Observatory No. 58). It gives with the interval 0.1 in M both $\tan\frac{v}{2}$ and $\tan^2\frac{v}{2}$ with 5 decimals and goes to $M = 300$ (corresponding to about $v = 120°$). Two years later a table by Subbotin (Publ. of the Tashkent Observatory vol. II) was published, which gives $\tan\frac{v}{2}$ and $\tan^2\frac{v}{2}$ with seven decimals and the same interval in M till $M = 144$ and with six decimals and an interval 10 times greater from $M = 142$ till $M = 300$. Consequently this table has got the same extension as the table given by B. Strömgren.

When q is not appreciably smaller than 1, these tables will be quite satisfactory, especially the table by B. Strömgren, which has constant interval through the whole table (and a number of decimals suitable for ordinary purposes). But when q is very small, observations of the comet may ordinarily be obtained far beyond the region, which is covered by these tables. If for instance $q = 0.1$, the limit of the tables is reached already 10 days after perihelion.

The present table gives $\tan\frac{v}{2}$ for values of M between 275 and 4515. For various reasons given below the table has not been computed for equidistant values of the argument M, but for equidistant values of $\tan\frac{v}{2}$. M has always been computed with 5 figures, corresponding to 5 significant figures in q. As the table is only used for small values of q, this usually corresponds to about the practical obtainable accuracy and will in any case be more than sufficient for the computation of an ordinary search-ephemeris, for which purpose the table mainly is meant.

The error in the value of $\tan\frac{v}{2}$ found through the table will occur in the 5^{th} decimal, and in the main part of the table it will not exceed 2 units. In a few exceptional cases immediately after $M = 1000$ it rises to 4—5 units. At the end of the table a higher accuracy is once more reached.

As mentioned above the quantities $\tan\frac{v}{2}$ and $1 - \tan^2\frac{v}{2}$ appear in the computation of ephemerides multiplied by q. If we assume q to be known to 5 significant figures, an error in the 5^{th} decimal (6^{th} figure) in $\tan\frac{v}{2}$ is irrelevant. Accordingly it is permissible that the second difference at a certain place in the table reaches 5 units in the last figure of M, in which cases the error through linear interpolation may rise to 0.6 units of the last figure. In any case the table gives a considerable higher accuracy than that obtained through 5-figure logarithmic computation.

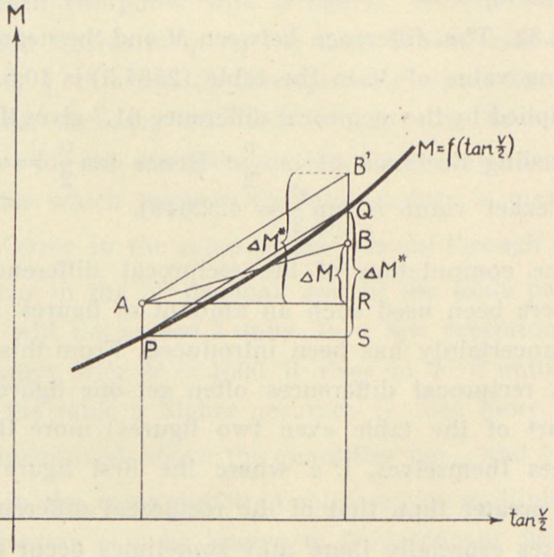
Instead of the differences of the M-values the reciprocal differences have been given to facilitate the interpolation, so that the otherwise necessary division has been transformed into a multiplication. In this manner the computation to be performed is the same as in a usual interpolation in a table with equidistant values of the argument. To avoid zeroes in front of the significant figures, the tabulated reciprocal differences are $\frac{1000}{\triangle M}$, so that the result is obtained in units of the 5^{th} decimal of $\tan\frac{v}{2}$.

Example. $M = 2575.0$. — The first figures in $\tan\frac{v}{2}$ are 4.32. The difference between M and the nearest preceeding value of M in the table (2564.5) is 10.5, which multiplied by the reciprocal difference 61.7 gives the three succeeding decimals of $\tan\frac{v}{2}$. Hence $\tan\frac{v}{2} = 4.32648$ (the exact value is $\tan\frac{v}{2} = 4.32649$).

In the computation of the reciprocal differences has everywhere been used such an amount of figures that no further uncertainty has been introduced. From this results that the reciprocal differences often get one figure (for a short part of the table even two figures) more than the differences themselves, i. e. where the first figure of the latter is greater than that of the reciprocal differences. In these cases especially there may sometimes occur remarkable irregular variation as seen for instance in the following part of the table:

M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$
1377.4		3.42
	96.2	
1387.8		3.43
	94.3	
1398.4		3.44
	95.2	
1408.9		3.45
	93.5	
1419.6		3.46

The tabulated reciprocal differences are as mentioned $\frac{1000}{\triangle M}$, where $\triangle M$ means the difference corresponding to the tabulated 5-figure values of M. It might seem reasonable to carry out a smoothing and to replace $\triangle M$ by the corresponding exact value $\triangle M^*$ and to tabulate $\frac{1000}{\triangle M^*}$ as reciprocal differences. But as is easily seen this means a di-



minished accuracy in the interpolation. In the plate the errors introduced by the rounding off of the last figure in two successive values of M are represented by PA and QB. Hence $SQ = RB' = \triangle M^*$ and $RB = \triangle M$.

If the above mentioned smoothing is performed it means that the interpolation is carried out along the line AB'. In the other case the interpolation is done along the line AB, which evidently gives the best approximation. In both cases a possibility for errors rising to a unit in the last figure of M certainly exists (as in all interpolations); but while in the first case this possibility exists in the whole length of the interval, it is in the latter instance only found in the immediate neighbourhood of the ends A and B of the interval. Hence the irregular variation of $\frac{1000}{\triangle M}$ has been retained.

Through tabulation using equidistant values of the function and reciprocal differences various advantages are obtained, the most important being the smaller size of the table and its greater uniformity. This method makes it possible to retain the same interval of the function through the whole table, while a constant interval of argument would mean an immense increase in the size of the table.

Furthermore the interpolation in a table of this kind is more simple than an ordinary interpolation, because the value of $\tan\frac{v}{2}$ with its two first decimals may be written down immediately, the interpolation then directly supplies the following decimals, and after the performance of the interpolation no necessity exists for returning to the table. This is a common advantage in tables of this kind. It might be mentioned that in our case the use of an equidistant interval of the function has simplified the computation of the table in a considerable degree.

If $\tan^2\frac{v}{2}$ was wanted with M as an argument, it would require a special table. No essential advantage would be gained, because the process of squaring a number carried out on a calculating machine hardly requires more time, than the corresponding process of finding $\tan^2\frac{v}{2}$ in a table.

Observatory Copenhagen
1931 October 12.

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M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$	M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$	M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$
274.40	311.5	1.70	404.62	232.6	2.05	576.14	179.2	2.40
277.61	308.6	1.71	408.92	231.5	2.06	581.72	178.3	2.41
280.85	305.8	1.72	413.24	228.8	2.07	587.33	176.7	2.42
284.12	304.0	1.73	417.61	227.8	2.08	592.99	175.4	2.43
287.41	300.3	1.74	422.00	225.7	2.09	598.69	174.5	2.44
290.74	298.5	1.75	426.43	223.7	2.10	604.42	173.0	2.45
294.09	295.0	1.76	430.90	222.2	2.11	610.20	171.8	2.46
297.48	293.3	1.77	435.40	220.8	2.12	616.02	170.6	2.47
300.89	290.7	1.78	439.93	218.8	2.13	621.88	169.5	2.48
304.33	288.2	1.79	444.50	216.9	2.14	627.78	168.4	2.49
307.80	285.7	1.80	449.11	215.5	2.15	633.72	167.2	2.50
311.30	283.3	1.81	453.75	214.1	2.16	639.70	166.1	2.51
314.83	280.9	1.82	458.42	212.3	2.17	645.72	165.0	2.52
318.39	278.6	1.83	463.13	210.5	2.18	651.78	163.7	2.53
321.98	276.2	1.84	467.88	209.2	2.19	657.89	162.9	2.54
325.60	274.0	1.85	472.66	207.5	2.20	664.03	161.6	2.55
329.25	271.0	1.86	477.48	205.8	2.21	670.22	160.5	2.56
332.94	269.5	1.87	482.34	204.5	2.22	676.45	159.2	2.57
336.65	267.4	1.88	487.23	202.8	2.23	682.73	158.5	2.58
340.39	264.6	1.89	492.16	201.6	2.24	689.04	157.2	2.59
344.17	263.2	1.90	497.12	199.6	2.25	695.40	156.2	2.60
347.97	260.4	1.91	502.13	198.4	2.26	701.80	155.0	2.61
351.81	258.4	1.92	507.17	197.2	2.27	708.25	154.3	2.62
355.68	256.4	1.93	512.24	195.3	2.28	714.73	153.1	2.63
359.58	254.5	1.94	517.36	194.2	2.29	721.26	152.0	2.64
363.51	252.5	1.95	522.51	192.7	2.30	727.84	151.3	2.65
367.47	250.0	1.96	527.70	191.2	2.31	734.45	149.9	2.66
371.47	248.1	1.97	532.93	190.1	2.32	741.12	149.3	2.67
375.50	246.3	1.98	538.19	188.3	2.33	747.82	148.1	2.68
379.56	244.5	1.99	543.50	187.3	2.34	754.57	147.3	2.69
383.65	242.1	2.00	548.84	185.9	2.35	761.36	146.2	2.70
387.78	240.4	2.01	554.22	184.5	2.36	768.20	145.3	2.71
391.94	238.1	2.02	559.64	183.2	2.37	775.08	144.3	2.72
396.14	237.0	2.03	565.10	181.8	2.38	782.01	143.5	2.73
400.36	234.7	2.04	570.60	180.5	2.39	788.98	142.5	2.74
404.62		2.05	576.14		2.40	796.00		2.75

M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$	M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$	M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$
796.00	141.6	2.75	1071.2	113.6	3.10	1408.9	93.5	3.45
803.06	140.6	2.76	1080.0	113.6	3.11	1419.6	93.5	3.46
810.17	139.9	2.77	1088.8	113.6	3.12	1430.3	93.5	3.47
817.32	138.9	2.78	1097.6	112.4	3.13	1441.0	92.6	3.48
824.52	138.1	2.79	1106.5	111.1	3.14	1451.8	91.7	3.49
831.76	137.2	2.80	1115.5	111.1	3.15	1462.7	91.7	3.50
839.05	136.2	2.81	1124.5	109.9	3.16	1473.6	90.9	3.51
846.39	135.5	2.82	1133.6	109.9	3.17	1484.6	90.9	3.52
853.77	134.6	2.83	1142.7	109.9	3.18	1495.6	90.1	3.53
861.20	133.7	2.84	1151.8	108.7	3.19	1506.7	89.3	3.54
868.68	133.0	2.85	1161.0	107.5	3.20	1517.9	89.3	3.55
876.20	132.1	2.86	1170.3	107.5	3.21	1529.1	88.5	3.56
883.77	131.2	2.87	1179.6	106.4	3.22	1540.4	88.5	3.57
891.39	130.4	2.88	1189.0	106.4	3.23	1551.7	87.7	3.58
899.06	129.7	2.89	1198.4	105.3	3.24	1563.1	87.7	3.59
906.77	128.9	2.90	1207.9	105.3	3.25	1574.5	87.0	3.60
914.53	128.0	2.91	1217.4	104.2	3.26	1586.0	86.2	3.61
922.34	127.4	2.92	1227.0	103.1	3.27	1597.6	86.2	3.62
930.19	126.4	2.93	1236.7	103.1	3.28	1609.2	85.5	3.63
938.10	125.8	2.94	1246.4	103.1	3.29	1620.9	85.5	3.64
946.05	125.0	2.95	1256.1	102.0	3.30	1632.6	84.7	3.65
954.05	124.2	2.96	1265.9	101.0	3.31	1644.4	84.0	3.66
962.10	123.5	2.97	1275.8	101.0	3.32	1656.3	84.0	3.67
970.20	122.9	2.98	1285.7	101.0	3.33	1668.2	83.3	3.68
978.34	122.0	2.99	1295.6	99.0	3.34	1680.2	82.6	3.69
986.54	121.2	3.00	1305.7	100.0	3.35	1692.3	82.6	3.70
994.79	120.6	3.01	1315.7	98.0	3.36	1704.4	82.6	3.71
1003.08	120.5	3.02	1325.9	98.0	3.37	1716.5	81.3	3.72
1011.4	119.0	3.03	1336.1	98.0	3.38	1728.8	81.3	3.73
1019.8	117.6	3.04	1346.3	97.1	3.39	1741.1	81.3	3.74
1028.3	117.6	3.05	1356.6	96.2	3.40	1753.4	80.6	3.75
1036.8	117.6	3.06	1367.0	96.2	3.41	1765.8	80.0	3.76
1045.3	116.3	3.07	1377.4	96.2	3.42	1778.3	80.0	3.77
1053.9	116.3	3.08	1387.8	94.3	3.43	1790.8	79.4	3.78
1062.5	114.9	3.09	1398.4	95.2	3.44	1803.4	78.7	3.79
1071.2		3.10	1408.9		3.45	1816.1		3.80

M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$	M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$	M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$
1816.1	78.7	3.80	2299.8	66.7	4.15	2867.1	57.1	4.50
1828.8	78.1	3.81	2314.8	66.2	4.16	2884.6	56.8	4.51
1841.6	77.5	3.82	2329.9	65.8	4.17	2902.2	56.5	4.52
1854.5	77.5	3.83	2345.1	65.8	4.18	2919.9	56.5	4.53
1867.4	76.9	3.84	2360.3	65.4	4.19	2937.6	56.2	4.54
1880.4	76.9	3.85	2375.6	65.4	4.20	2955.4	55.9	4.55
1893.4	76.3	3.86	2390.9	64.5	4.21	2973.3	55.9	4.56
1906.5	75.8	3.87	2406.4	64.5	4.22	2991.2	55.2	4.57
1919.7	75.8	3.88	2421.9	64.5	4.23	3009.3	55.2	4.58
1932.9	75.2	3.89	2437.4	63.7	4.24	3027.4	54.9	4.59
1946.2	74.6	3.90	2453.1	63.7	4.25	3045.6	54.9	4.60
1959.6	74.6	3.91	2468.8	63.3	4.26	3063.8	54.6	4.61
1973.0	74.1	3.92	2484.6	63.3	4.27	3082.1	54.1	4.62
1986.5	74.1	3.93	2500.4	62.9	4.28	3100.6	54.3	4.63
2000.0	73.5	3.94	2516.3	62.5	4.29	3119.0	53.8	4.64
2013.6	73.0	3.95	2532.3	62.1	4.30	3137.6	53.8	4.65
2027.3	72.5	3.96	2548.4	62.1	4.31	3156.2	53.5	4.66
2041.1	72.5	3.97	2564.5	61.7	4.32	3174.9	53.2	4.67
2054.9	71.9	3.98	2580.7	61.3	4.33	3193.7	52.9	4.68
2068.8	71.9	3.99	2597.0	61.3	4.34	3212.6	52.9	4.69
2082.7	71.4	4.00	2613.3	61.0	4.35	3231.5	52.4	4.70
2096.7	70.9	4.01	2629.7	60.6	4.36	3250.6	52.4	4.71
2110.8	70.9	4.02	2646.2	60.2	4.37	3269.7	52.4	4.72
2124.9	70.4	4.03	2662.8	60.2	4.38	3288.8	51.8	4.73
2139.1	69.9	4.04	2679.4	59.9	4.39	3308.1	51.8	4.74
2153.4	69.9	4.05	2696.1	59.5	4.40	3327.4	51.5	4.75
2167.7	69.4	4.06	2712.9	59.5	4.41	3346.8	51.3	4.76
2182.1	69.0	4.07	2729.7	59.2	4.42	3366.3	51.0	4.77
2196.6	68.5	4.08	2746.6	58.8	4.43	3385.9	51.0	4.78
2211.2	68.5	4.09	2763.6	58.5	4.44	3405.5	50.5	4.79
2225.8	68.5	4.10	2780.7	58.5	4.45	3425.3	50.5	4.80
2240.4	67.6	4.11	2797.8	57.8	4.46	3445.1	50.3	4.81
2255.2	67.6	4.12	2815.1	58.1	4.47	3465.0	50.3	4.82
2270.0	67.1	4.13	2832.3	57.5	4.48	3484.9	49.8	4.83
2284.9	67.1	4.14	2849.7	57.5	4.49	3505.0	49.8	4.84
2299.8		4.15	2867.1		4.50	3525.1		4.85

M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$	M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$	M	$\frac{1000}{\triangle M}$	$\tan\frac{v}{2}$
3525.1	49.5	4.85	3836.5	46.5	5.00	4166.5	44.1	5.15
3545.3	49.3	4.86	3858.0	46.5	5.01	4189.2	44.1	5.16
3565.6	49.3	4.87	3879.5	46.5	5.02	4211.9	43.7	5.17
3585.9	48.8	4.88	3901.0	46.1	5.03	4234.8	43.7	5.18
3606.4	48.8	4.89	3922.7	45.9	5 04	4257.7	43.5	5.19
3626.9	48.5	4.90	3944.5	45.9	5.05	4280.7	43.3	5.20
3647.5	48.3	4.91	3966.3	45.7	5.06	4303.8	43.1	5.21
3668.2	48.3	4.92	3988.2	45.5	5.07	4327.0	43.1	5.22
3688.9	47.8	4.93	4010.2	45.2	5.08	4350.2	42.7	5.23
3709.8	47.8	4.94	4032.3	45.2	5.09	4373.6	42.7	5.24
3730.7	47.6	4.95	4054.4	44.8	5.10	4397.0	42.4	5.25
3751.7	47.4	4.96	4076.7	44.8	5.11	4420.6	42.4	5.26
3772.8	47.2	4.97	4099.0	44.6	5.12	4444.2	42.2	5.27
3794.0	47.2	4.98	4121.4	44.4	5.13	4467.9	42.0	5.28
3815.2	46.9	4.99	4143.9	44.2	5.14	4491.7	42.0	5.29
3836.5		5.00	4166.5		5.15	4515.5		5.30

Færdig fra Trykkeriet den 8. Februar 1932.